Sheafhom

A package for sparse linear algebra and algebraic topology

Mark McConnell
Overview

• Package for large-scale mathematical computations
• Front end for algebraic topology and number theory
• Back end solves large sparse systems of linear equations over the integers
History

- Sheafhom 1.x, 1993-99, CLtL1/2
- Sheafhom 2.0, 2001-04, Java
- Sheafhom 2.1, 2004-05, ANSI CL
- At http://www.lispwire.com and www.geocities.com/mmcconnell17704
Topology

• Ash, Gunnells, —: cohomology of arithmetic subgroups. Compute Hecke eigenvalues, look for attached Galois representations, as examples of the Langlands program.

• Dimensions 3, 4, 6, embedded in up to 10.

• Today, stick to 2 dimensions.
Two-Complexes

- A *two-complex* is a space made by gluing triangles together along their edges and vertices.
- Or any space *homeomorphic* to one of these.
Sphere $S^2$
Barycentric Subdivision
Torus $T^2$
Klein Bottle (non-orientable)
Homology

• Form a group of $i$-dimensional cycles ($i$-dimensional loops) on a space $X$.
• Two cycles are equivalent if they are the boundary of an $(i+1)$-dimensional object.
• The result is the $i$-th homology $H_i(X)$.
• I’ll speak of the rank of $H_i$, the number of independent generators.
$H_0$ Measures Connectedness

- $H_0$ has rank 1
$H_1 = \text{Essential Loops}$

- $H_1$ has rank 0
$H_I = \text{Essential Loops}$

- $H_I$ has rank 2
Compute Hi With Boundary Matrices

\[
\begin{pmatrix}
1 & 1 & 0 & 1 & 0 & 0 \\
-1 & 0 & 1 & 0 & 1 & 0 \\
0 & -1 & -1 & 0 & 0 & 1 \\
0 & 0 & 0 & -1 & -1 & -1 \\
\end{pmatrix}
\]

\[
((1 \ 2 \ 3) \ (0 \ 2 \ 3) \ (0 \ 1 \ 3) \ (0 \ 1 \ 2))
\]
$H_1$ on non-orientable spaces

- One loop goes all around the outside
- Red loop $\neq$ green loop
- But $2 \times (\text{red loop}) = (\text{red + green}) = (\text{do something, then undo}) = 0$.
- $Torsion$: $2 \times x = 0$ yet $x \neq 0$.
- $H_1 = \mathbb{Z} + \mathbb{Z}/2\mathbb{Z}$.